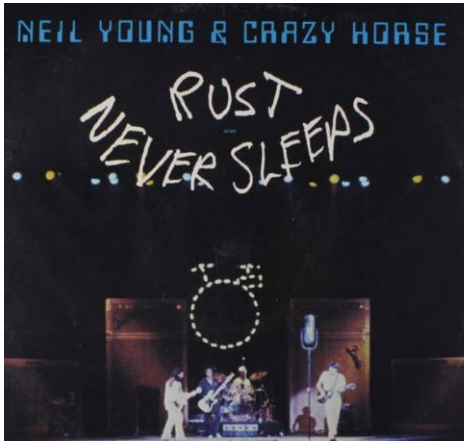
DOE Next Level Methods



Split Plot Designs and Navy Corrosion Testing

Experiments with Random Factors

- Random effects (usually categorical) are appropriate when the number of levels is large and it is difficult, undesirable, or impractical to adequately define the factor levels
  - A few factor levels are chosen at random from a larger population of potential levels
  - Inference is about the entire population of levels
- Applications
  - Measurement system studies – gauge R&R
  - Manufacturing variability – hardware serial number
  - Operators
    - 2 operators in your test are the only 2 that would ever use the system = fixed effect
    - 2 operators in your test are selected at random = random effect

Randomization

- 3 big design principles:

randomization, replication, and blocking

- What happens when an experimenter cannot completely randomize the runs?
  - We cannot run the experiments in random order because one or more settings are "hard-to-change."
  - The experiments have both "hard-to-change" and "easy-to-change" factors.
  - Assumptions used in the analysis are no longer valid unless the design accounts for this restriction on randomization.

Split-Plot Design

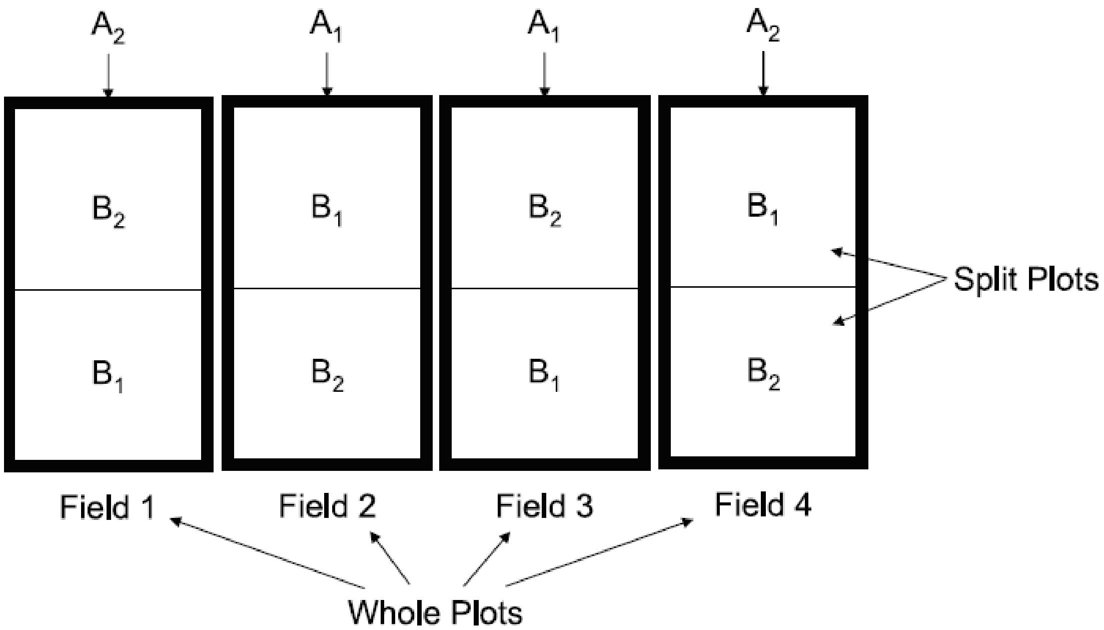


FIGURE 1. Split Plot Agricultural Layout. (Factor *A* is the whole-plot factor and factor *B* is the split-plot factor.)

Reference: "*Split-Plot: What, Why, and How*" by Jones and Nachtsheim. *Journal of Quality Technology*. Vol. 41:No. 4. October 2009.

Split-Plot Design

- A *split-plot design* is used to account for this restriction on randomization.
  - Hard-to-change factors are placed in whole plots.
  - Easy-to-change factors are placed in split or subplots within each whole plot.
  - The two levels of randomization are the following:
    - one to the assignment of whole-plot treatments
    - one to the assignment of subplot treatments

  Therefore, the model for these designs has two components of variation.

Reference: "*Split-Plot: What, Why, and How*" by Jones and Nachtsheim. *Journal of Quality Technology*. Vol. 41:No. 4. October 2009.

Split-Plot Experiments

- "In simple terms, a split-plot experiment is a blocked experiment, where the blocks themselves serve as experimental units for a subset of the factors. Thus, there are two levels of experimental units. The blocks are referred to as *whole plot*s, while the experimental units within blocks are called *split plots*, *split units*, or *subplots*. Corresponding to the two levels of experimental units are two levels of randomization. One randomization is conducted to determine the assignment of block-level treatments to whole plots. Then, as always in a blocked experiment, a randomization of treatments to split-plot experimental units occurs within each block or whole plot."

- Jones and Nachtsheim, "Split-Plot Designs: What, Why, and How"

Split-Plot Design – Example

- Consider a materials science experiment.
  - Response: Strength
  - Factors: **Time**, **Temp**, **Rate**, and **pH**

- However, because this is done on a piece of equipment that has a "set" **Rate** and **Temp**, both of these factors are **hard-to-change** factors. Both Time and Angle are easy-to-change factors.

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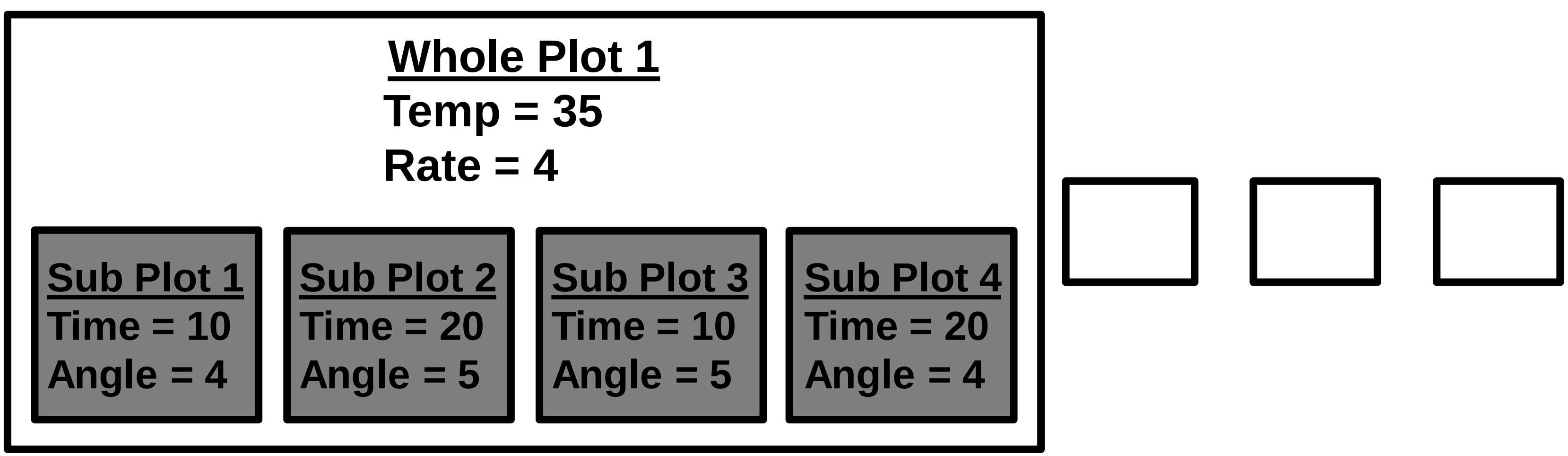
Whole plots

- In a split-plot design, the levels of the two hard-to-change factors (**Rate** and **Temp**) are randomly assigned to whole plots.
  - Four whole plots
- Each whole plot contains a treatment combination of both hard-to-change factors.

Whole Plot 1	**Whole Plot 2**	**Whole Plot 3**	**Whole Plot 4**
Temp = 35	**Temp = 37**	**Temp = 37**	**Temp = 35**
Rate = 4	**Rate = 4**	**Rate = 2**	**Rate = 2**

Subplots

- In a split-plot design, the levels of the two easy-to-change factors (**Time** and **Angle**) are randomly assigned to each whole plot.
  - Four subplots are within each whole plot.
  - Each subplot contains a treatment combination of both easy-to-change factors.



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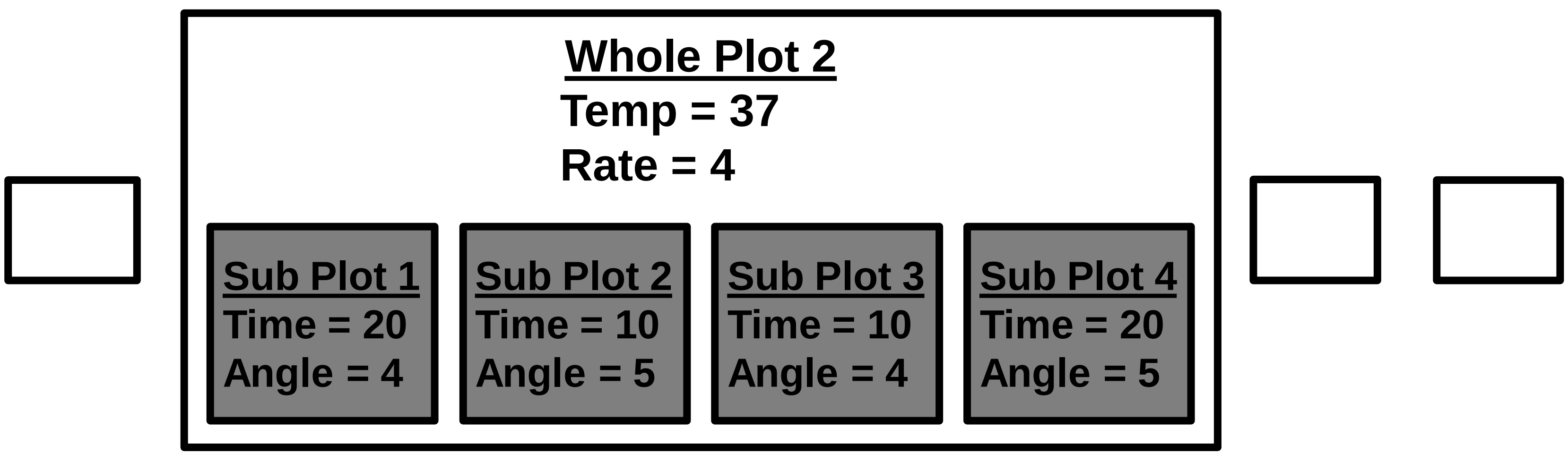

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Subplots

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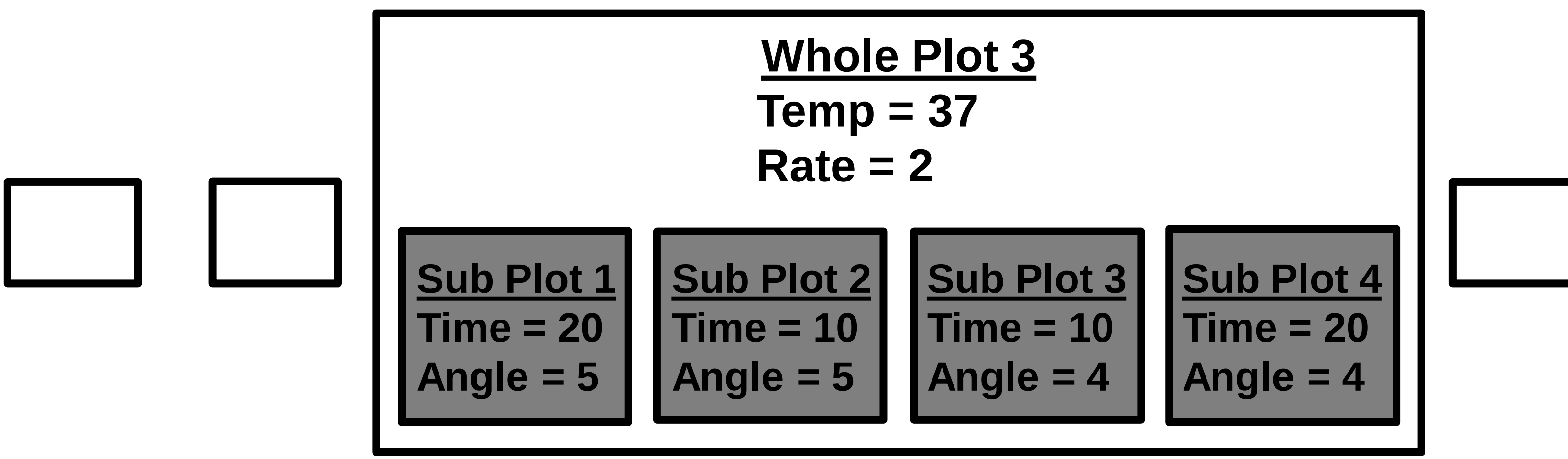

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Subplots

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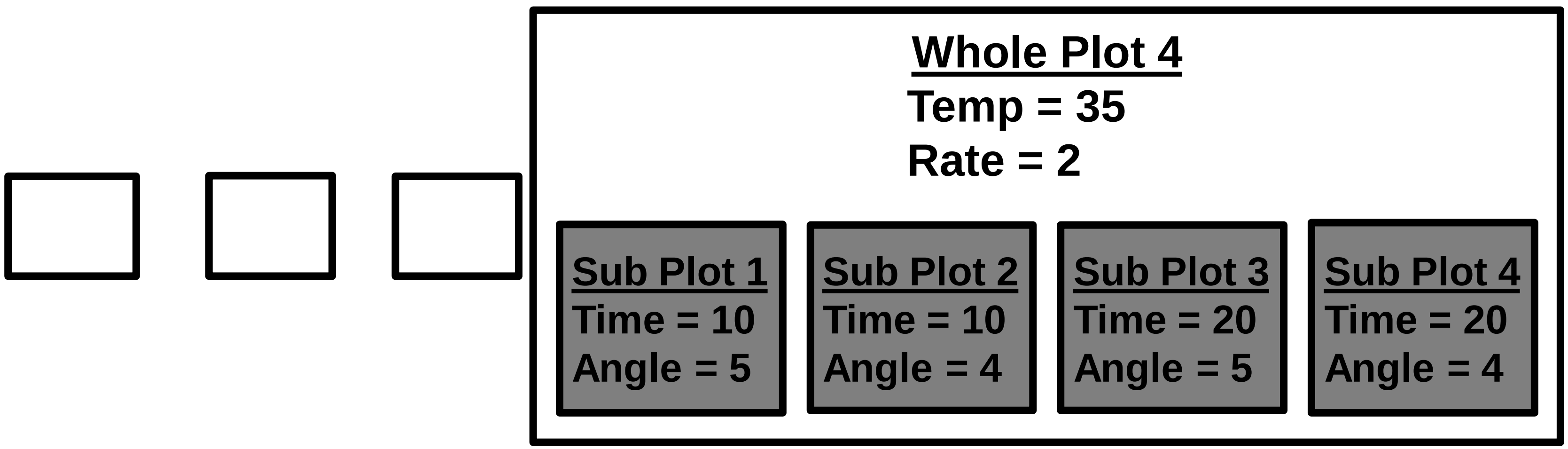

Split-Plot Design – Example

- Assume that the scientist designs the same experiment as before.
  - Response: Productivity
  - Factors: **Time**, **Temp**, **Rate**, and **Angle**

However, because this is done on a piece of equipment that has a “set” **Rate** and **Temp**, both of these factors are *hard-to-change* factors. Both **Time** and **Angle** are *easy-to-change* factors.

Design

Run	Whole Plots	Temp	Rate	Time	Angle
1	1	35	4	20	4
2	1	35	4	20	5
3	1	35	4	10	5
4	1	35	4	10	4
5	2	35	2	20	4
6	2	35	2	20	5
7	2	35	2	10	4
8	2	35	2	10	5
9	3	37	4	10	4
10	3	37	4	20	4
11	3	37	4	10	5
12	3	37	4	20	5
13	4	37	2	10	5
14	4	37	2	10	4
15	4	37	2	20	4
16	4	37	2	20	5

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Split-Plot Design

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Tutorial: Industrial Split-plot Experiments

Scott M. Kowalski
Minitab Inc. State College, PA

Peter A. Parker
NASA Langley Research Center, Hampton, VA

G. Geoffrey Vining
Virginia Polytechnic Institute and State University, Blacksburg, VA

ABSTRACT Many industrial experiments involve two types of factors: those that are hard-to-change and those that are easy-to-change (ETC). Hard-to-change (HTC) factors have levels that are difficult and/or expensive to change. As a result, the experimenter would prefer to run the experiment in such a manner as to minimize the number of times that he/she must change the levels of these factors. Unfortunately, it is precisely the changing of these levels that provides the information about the effects of the HTC factors. Consequently, when we minimize the number of times we change the levels of these factors, we also minimize the relevant information about their effects.

This paper summarizes the structure and the analysis of industrial split-plot experiments. The purpose of this article is to teach practitioners how to identify split-plot experimental conditions, how to run the experiment efficiently, and then how to analyze the results. The article illustrates both first-order and second-order experiments. The first four sections provide a basic background on experimental design and an introduction to first-order split-plot experiments. The remainder of this article contains more advanced topics dealing with second-order, split-plot experiments.

KEYWORDS design of experiments, response surface methodology, split-plot experiments

Split-Plot Design

We use an example to illustrate the design and analysis of a split-plot design for fitting a first-order model with interactions. Consider an experiment involving the strength of plastic. The four factors identified as potentially important are: (a) baking temperature, (b) additive percentage, (c) agitation rate, and (d) processing time. Each factor has two levels: low = −1 and high = 1. The levels are left in coded units for proprietary reasons. The experiment first makes a mold from the additive percentage, the agitation rate, and the processing time. Then the mold is baked at one of the temperatures. Finally, the strength of the plastic is measured.

TABLE 5 Data for the Plastic Example

Whole-plot	Temp	Additive	Rate	Time	Strength	Whole-plot mean
1	1	−1	1	1	68.5	
1	1	1	−1	1	66.8	
1	1	−1	−1	−1	58.5	
1	1	1	1	1	70.8	62.94
1	1	−1	1	−1	61.3	
1	1	1	−1	−1	51.9	
1	1	−1	−1	1	59.5	
1	1	1	1	−1	66.2	
2	−1	1	−1	−1	57.4	
2	−1	1	−1	1	57.5	
2	−1	−1	1	−1	56.5	
2	−1	1	1	1	63.9	57.81
2	−1	−1	1	1	56.4	
2	−1	1	1	−1	58.1	
2	−1	−1	−1	1	53.2	
2	−1	−1	−1	−1	59.5	
3	−1	−1	−1	−1	66.6	
3	−1	−1	−1	1	63.9	
3	−1	1	1	−1	62.6	
3	−1	1	1	1	63.2	62.93
3	−1	−1	1	−1	56.1	
3	−1	1	−1	1	63.3	
3	−1	−1	1	1	62.7	
3	−1	1	−1	−1	65.0	
4	1	−1	−1	−1	59.5	
4	1	1	1	−1	64.0	
4	1	−1	1	1	68.0	
4	1	1	−1	−1	65.6	64.34
4	1	−1	1	−1	58.6	
4	1	1	1	1	73.3	
4	1	1	−1	1	61.5	
4	1	−1	−1	1	64.2	

Source	df	SS	MS	F	p
Temp	1	85.478	85.478	1.52	0.343
WP Error	2	112.391	56.195	*	*
Add	1	45.363	45.363	4.64	0.044
Rate	1	41.178	41.178	4.21	0.054
Time	1	75.953	75.953	7.76	0.012
Add*Rate	1	27.938	27.938	2.86	0.107
Add*Time	1	2.940	2.940	0.30	0.590
Rate*Time	1	43.945	43.945	4.49	0.047
Temp*Add	1	1.088	1.088	0.11	0.742
Temp*Rate	1	78.438	78.438	8.02	0.011
Temp* Time	1	62.440	62.440	6.38	0.021
SP Error	19	185.858	9.782		
Total	31	763.010			

*Not necessary to test.

Plastics Example

Split-Plot Design

Plastics Example

Completely Randomized Analysis

Summary of Fit

RSquare	0.450173
RSquare Adj	0.344437
Root Mean Square Error	4.016902
Mean of Response	62.00313
Observations (or Sum Wgts)	32

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	5	343.48656	68.6973	4.2575
Error	26	419.52312	16.1355	Prob > F
C. Total	31	763.00969		0.0058*

Parameter Estimates CRD

Term	Estimate	Std Error	t Ratio	Prob>|t|	VIF
Intercept	62.003125	0.710095	87.32	<.0001*	.
Temp	1.634375	0.710095	2.30	0.0296*	1
Rate	1.134375	0.710095	1.60	0.1222	1
Time	1.540625	0.710095	2.17	0.0394*	1
Temp*Rate	1.565625	0.710095	2.20	0.0365*	1
Temp*Time	1.396875	0.710095	1.97	0.0599	1

Split Plot Analysis

Summary of Fit

RSquare	0.709947
RSquare Adj	0.625348
Root Mean Square Error	3.146603
Mean of Response	62.00313
Observations (or Sum Wgts)	32

Parameter Estimates SPD

Term	Estimate	Std Error	DFDen	t Ratio	Prob>|t|	VIF
Intercept	62.003125	1.325181	2	46.79	0.0005*	.
Temp	1.634375	1.325181	2	1.23	0.3427	1
Additive	1.190625	0.596377	23	2.00	0.0579	1
Rate	1.134375	0.596377	23	1.90	0.0698	1
Time	1.540625	0.596377	23	2.58	0.0166*	1
Temp*Rate	1.565625	0.596377	23	2.63	0.0151*	1
Temp*Time	1.396875	0.596377	23	2.34	0.0282*	1

Random Effect Predictions

Term	BLUP	Std Error	DFDen	t Ratio	Prob>|t|
Whole Plots[1]	-0.558228	1.835249	1.696	-0.30	0.7942
Whole Plots[2]	-2.03853	1.835249	1.696	-1.11	0.3994
Whole Plots[3]	2.0385302	1.835249	1.696	1.11	0.3994
Whole Plots[4]	0.5582283	1.835249	1.696	0.30	0.7942

REML Variance Component Estimates

Random Effect	Var Ratio	Var Component	Std Error	95% Lower	95% Upper	Wald p-Value	Pct of Total
Whole Plots	0.4921896	5.6017527	7.0369304	-8.190377	19.393883	0.4260	32.984
Residual		11.381291	3.3561595	6.8749934	22.395391		67.016
Total		16.983043	7.6135567	8.2793861	52.488028		100.000

Estimate of Error in Split Plots

- If the experiment is erroneously analyzed as a completely randomized design (CRD), the sources of error can lead to incorrect conclusions on the active variable set
- CRD will underestimate the error for the whole plot (hard to change) error; therefore these may be declared active when they are not (false positive)
- CRD may overestimate the split plot or subplot error and not detect active easy to change factors (false negative)
- If the experiment is not set up as a split plot design, but “clever” testers decide to run the test more efficiently by not changing factors you can still analyze correctly as a split plot.